TECHNICAL MEMORANDUM

November 28, 2019

TO: UBC MINE331 CLASS
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SUBJECT: SAG MILL + BALL MILL CIRCUIT SIZING

Summary:

Most large tonnage mineral processing facilities consist of a primary SAG milling and secondary ball milling grinding circuit. There are three power-based methodologies for sizing SAG mills that are widely used:

- Bond work index based methods;
- Australian "drop weight test" based methods;
- The SPI/SAGdesign methods.

Each methodology involves a series of grindability laboratory tests and a suite of equations used for mill power and throughput predictions. This presentation deals only with the Bond work index suite.

The desired outcome of all of these methods is an estimate of the grinding specific energy required to reduce a rock from a "F_{80}" size to a "P_{80}" size. This energy is expressed as kW·h/tonne, where kW is the grinding energy required "at the mill shell" multiplied by the inverse of throughput (h/tonne). This energy requirement (for a particular F_{80} and P_{80}) is a property of the rock and any difference in specific energy of different comminution devices is (largely) reflective of the efficiency of the grinding device.

Once the specific energy is determined, the amount of grinding power needed (at the mill shell) is found by multiplying the specific energy by the desired throughput. Then a motor size and mill geometry is chosen that will satisfy the power demand at the mill shell.

Types of Models Used in Comminution

Many different types of modelling are used in comminution system design and optimization; they can usually be classified into these categories:

Power draw models. Predict the power draw of a mill for a given charge geometry and rotation speed.

Specific energy models. These assume 'standard' particle size distributions and require only a single size point (usually 80% passing) to characterize an entire particle size distribution. Simple and mostly linear models, they can be run very quickly using computers.

Population balance models. These track flows of individual size classes separately and operate well in situations where "non-standard" particle size distributions are being used. Models tend to be complex; mostly suitable for optimization of an existing mill, but not for preliminary design work.

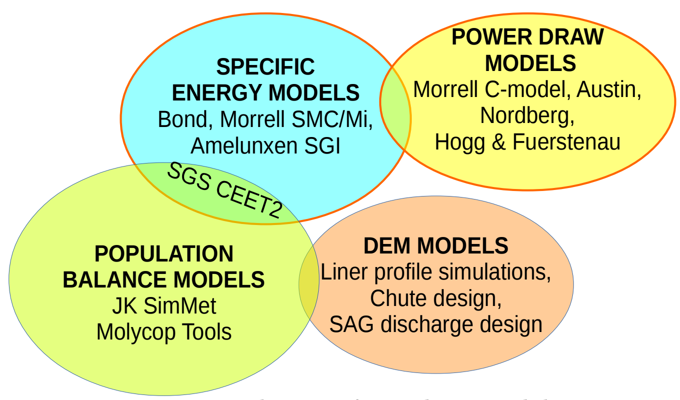

Figure 1: Classes of grinding models

Discrete element models (DEM). These use fundamental physics to model the motion of simulated particles in a gravity field, and use complex collision calculations to predict the motion of particles as they move within, for example, a turning mill. Very complex and are suited only to detailed design of components of a milling system, such as the liner face angle and height.

The Bond Work Index

Before the 1950's the grinding industry had no ability to scale-up grinding results from laboratory to industrial mill sizes, or to reliably estimate the specific energy when either a feed or product size changed. The industry had been measuring specific energy consumption for decades, but didn't really know how to make reliable predictions involving them[1].

Fred Bond worked at the Allis Chalmers mineral processing equipment company in the middle part of the 20th century. He had access to one of the best equipped grinding laboratories in the world, and access to data from hundreds of operating plants all over North America. His employer, Allis Chalmers, was one of the largest suppliers of grinding equipment. Allis Chalmers had a huge stake in selling "correctly" sized grinding equipment, so they were motivated to do research find better ways to scale up laboratory grindability results into full-sized mills.

What Bond developed was a simple formula that expressed specific energy consumption with three parameters: the feed size, product size, and a grindability measurement of the ore. This measurement became known as a "work index"[2]. Bond's formula is usually presented in this form:

$$E = 10 \times Wi \left(\frac{1}{\sqrt{P_{80}}} - \frac{1}{\sqrt{F_{80}}} \right) \quad (1)$$

where:

- E is the specific energy consumption, kWh/tonne;
- Wi is the work index, unitless, but applicable only to metric calculations;
- F_{80} is the 80% passing size of the feed, µm;
- P_{80} is the 80% passing size of the product, µm.

The work index laboratory measurement can be made with a variety of equipment. The most common is the Bond ball mill work index:

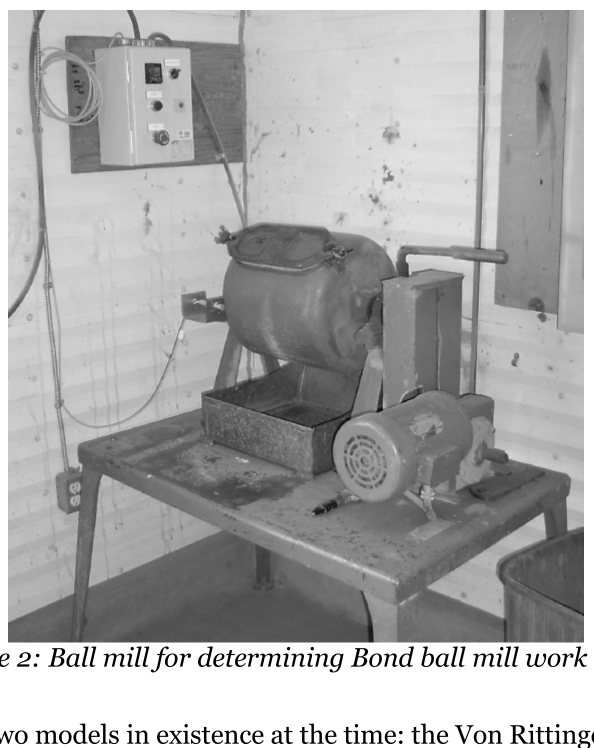

Figure 2: Ball mill for determining Bond ball mill work index

1 For the purists, there were two models in existence at the time: the Von Rittinger and Kick models. Neither of these gave results that matched the observations of specific energy consumption from tumbling mills.

2 **F.C. Bond**; *The Third Theory Of Comminution,* Transactions AIME 1952. Available from www.onemine.org

Many people refer to “the” Bond work index when they really mean the Bond *ball mill* work index. There are two other Bond work index measurements commonly used: the rod mill work index and the crushing (low-energy impact) work index.

Figure 3: Rod mill for determining Bond rod mill work index

So why are there three machines measuring three different versions of the same index? Because the nature of rock breakage changes with size. Coarse rock breakage is dominated by fractures, whereas medium-size rocks break through the rock matrix, and fine-size rocks break along and through mineral grains. To capture these three mechanisms, the Bond method makes use of three tests, each suited to a particular size range and type of breakage.

Table 1: Typical size ranges applicable to the Bond work index tests

	Feed Size, F_{80}	Product Size, P_{80}
Crushing work index, Wi_C	100 mm	10 mm
Rod mill work index, Wi_{RM}	10 mm	2100 µm
Ball mill work index, Wi_{BM}	2100 µm	100 µm

Determining Overall Circuit Specific Energy Consumption

To predict the cumulative breakage from a coarse size to a fine size requires doing three calculations using Equation 2, one for each of the three work index measurements and handing-off intermediate sizes per Table 1. But before doing that calculation, there are some correction factors that need to be applied.

Chet Rowland worked with Bond's original equations and found that some adjustments were needed in certain applications where machines were being operated outside of their range of highest efficiency. These “EF” factors are tabulated in many texts[1] and not all are used in particular calculations. Be aware that many old texts refer to work index on a short ton basis, and these must be converted to a metric tonne basis to perform modern calculations.

Of interest to this style of calculation are the Rowland EF4 and EF5 factors for ball milling. The following equations are for metric work index values and particle sizes in µm:

$$EF4_{BM}=\left[1+\frac{(0.907\times Wi_{BM}-7)}{\left(\frac{F_{80}}{P_{80}}\right)}\left(\frac{F_{80}}{4\,000\left(\frac{14.33}{Wi_{RM}}\right)^{0.5}}-1\right)\right] \quad (2)$$

1 C. Rowland. Chapter 23: *The Design and Installation of Comminution Circuits*. AIME 1982

$$EF5_{BM}=\left[\frac{P_{80}+10.3}{1.145\times P_{80}}\right] \quad (3)$$

The EF4 formula requires both the rod mill and ball mill work index (rod mill Wi is used to calculate the optimal feed size) and because this is a "single-stage ball mill" calculation, the F_{80} is actually the rod mill feed size (10,000 µm) from Table 1 and the P_{80} is the ball mill circuit product. The EF5 factor only applies below 75 µm to ball milling, and a value of 1 can be substituted above that P_{80}. Values of EF4 can only be greater than 1. If an EF4/5 value evaluates to less than 1, skip that factor and substitute 1 in its place. Combining the Rowland efficiency factors with the three-stage Bond equation gives this overall equation for grinding specific energy consumption from a primary crusher product size, F_{80}, to a ball mill cyclone overflow product size, P_{80}.

$$E=10\times Wi_C\left(\frac{1}{\sqrt{10000}}-\frac{1}{\sqrt{F_{80}}}\right)+\left[10\times Wi_{RM}\left(\frac{1}{\sqrt{2100}}-\frac{1}{\sqrt{10000}}\right)+10\times Wi_{BM}\left(\frac{1}{\sqrt{P_{80}}}-\frac{1}{\sqrt{2100}}\right)\times EF5_{BM}\right]\times EF4_{BM} \quad (4)$$

This energy is the "maximum efficiency" comminution energy that would be associated with highly efficient comminution machines: typically multi-stage crushing, rod milling and ball milling or a Noranda-style multi-stage crushing and single-stage ball milling circuit. Barratt, 1989, writes that a copper porphyry ore in a power-efficient circuit of SAG milling followed by ball milling should be 10% less efficient that the "maximum efficiency" grinding circuit (crushing, single-stage ball mill). Thus, the E_{total} of a SAG and ball mill circuit is:

$$E_{total}=1.10\times E \quad (5)$$

Partitioning the Total Energy Between SAG and Ball Mills

The Bond-based method applied by Barratt involves deducting a SAG mill specific energy consumption, E_{SAG}, from the overall circuit specific energy consumption, E_{total}, with the difference being the ball mill specific energy consumption, E_{bm}. One of the advantages of this approach is it accounts for the tendency for SAG mills to make more fines than a crusher or rod mill would in the same duty. These fines are passed to the ball mill sump and into the cyclone where they exit the circuit without actually appearing in the ball mill. This results in an apparent reduction in ball mill specific energy consumption versus what would be predicted using the straight Bond formula and the ball mill work index. This phenomenon is often called the *phantom cyclone* because of how some of the other modelling approached account for the extra fines.

The specific energy consumption for the SAG mill can be estimated using Barratt's 1979 formula (which was republished in the 1989 SAG conference[1]):

$$E_{SAG}=\left[10\times Wi_C\left(\frac{1}{\sqrt{P_C}}-\frac{1}{\sqrt{F_{80}}}\right)+10\times Wi_{RM}\left(\frac{1}{\sqrt{P_R}}-\frac{1}{\sqrt{P_C}}\right)\times EF4_{RM}+10\times Wi_{BM}\left(\frac{1}{\sqrt{110}}-\frac{1}{\sqrt{P_R}}\right)\times EF4_{BM}\times EF5\right]\times 1.25 \\ -10\times Wi_{BM}\left(\frac{1}{\sqrt{110}}-\frac{1}{\sqrt{T_{80}}}\right)\times EF4_{BM}\times EF_5 \quad (6)$$

The form of this equation is similar to Equation 4, except that it introduces a few new terms and it includes a factor of 1.25 that accounts for the difference in grinding efficiency of a SAG mill versus a ball mill. The original Barratt papers do not specify values to use for P_C and P_R, but we can use the same values as Table 1 of 10 000 µm and 2100 µm, respectively. The value T_{80} is the "synthetic" transfer size between the SAG mill and ball mill – a fines-corrected 80% passing size[2] of the SAG mill product (or the combined SAG mill product and crushed pebbles if the circuit is sending crushed pebbles to the ball mill circuit, a.k.a. SABC-B operation). The $EF4_{RM}$ for rod milling is used, calculated using Equation 7.

1 D.J. Barratt. *An Update On Testing, Scale-up and Sizing Equipment for Autogenous and Semi-Autogenous Grinding Circuits.* 1989 SAG Conference.

2 The actual transfer size you measure in a plant survey will be finer than this synthetic value. To determine the fines-corrected T_{80} from a survey value, perform a "phantom cyclone" or "reduced-recovery" calculation (see Barratt, 1989).

$$EF4_{RM} = \left[1 + \frac{(0.907 \times Wi_{RM} - 7)}{\left(\frac{F_{80}}{P_{80}}\right)} \left(\frac{F_{80}}{16000 \left(\frac{14.33}{Wi_{RM}}\right)^{0.5}} - 1 \right) \right] \quad (7)$$

The transfer size is normally set to a value in the range of 1 to 3 mm for a "SABC-A" circuit where the pebbles discharging the SAG mill are screened and crushed in a pebble crusher, then the crushed pebbles are returned to the SAG mill feed. The recommended design value is 2 mm for T_{80} in most calculations.

Pebble crushers are considered to be part of the SAG mill portion of the circuit in these calculations. Normally, the energy that pebble crushers input to the ore breakage is insignificant (on the order of 2% of the overall SAG+pebble crusher power) and can be neglected.

The SAG mill specific energy consumption is deducted from the total circuit specific energy consumption to get the ball mill specific energy consumption:

$$E_{bm} = E_{total} - E_{SAG} \quad (8)$$

Determine Mill and Motor Size

Knowing the SAG and ball mill specific energy consumption allows mill sizes to be calculated if a desired throughput rate is known. Note the units of specific energy: kW·h/tonne. This is power (kW) times the reciprocal of throughput (t/h).

The power in Bond calculations is the power as seen by the ore at the shell of the mill. This is often called the "power at the pinion" because smaller mills use a pinion and gear arrangement to transmit motor power to the mill. A gear and pinion have mechanical losses, meaning the actual motor output power will be greater than the grinding calculations which operate in "power at the pinion"[1].

The SAG mill motor will normally operate at 90% of its rated output power. Combining this design criteria with the pinion/gear efficiency (use 0.985 for a synchronous motor, pinion and gear; use 1.000 for gearless) provides the following equation to estimate the motor size for a gear-driven SAG mill:

$$\text{Total SAG Motor Power} = (E_{SAG} \times \text{tonnes/hour}) \div 0.985 \div 0.90 \quad (9)$$

The ball mill motor can be sized using a similar equation, but ball mills normally operate at a higher proportion of their rated power output, use 0.94. The ball mill gear/pinion efficiency is the same as for a SAG mill. For a gear-driven ball mill, use the following:

$$\text{Total Ball Mill Motor Power} = (E_{bm} \times \text{tonnes/hour}) \div 0.985 \div 0.94 \quad (10)$$

The largest motor sizes currently on the market with pinion and gear drives are in the 10 MW range (these would be a twin-motor arrangement with one 10 MW motor on either side of the mill). Gearless drives are available for over 20 MW motor output (23.5 MW is running at Esperanza in Chile and 28 MW drives are under construction for mines in Peru and Australia). If your motor power exceeds these sizes, split the duty into multiple mills of the same type.

Determining Mill Dimensions

Nominal mill diameters are specified inside the mill shell, before the liners are installed. Two types of mill length are commonly used in the industry.

- "Effective Grinding Length," or EGL, is the length of the mill cylinder where grinding can occur. This is generally the length from the inside of the mill feed end liner to the mill discharge grate. This is the preferred designation of process engineers.
- "Flange to Flange Length," or F/F, is the length of the cylinder of the mill as seen from the outside flange positions where the feed and discharge cones are bolted to the cylinder. This is the preferred designation for mechanical engineers.

1 See https://www.sagmilling.com/articles/1/view/?s=1 for more discussion of power measurements at different parts of a grinding mill electrical & control circuitry.

The detailed calculation methods for determination of mill dimensions to yield a particular power draw (at a mill shell) is beyond the scope of this Memorandum. Instead, two ways to size mills are offered:

1. Look in literature for a mill that has the same motor as you have calculated. ~~Plagiarize~~ Copy that design. Don't use any mills from Africa – they use a completely different design (low-aspect-ratio and high speed). Also, don't use an iron mine as a basis for a copper or gold project because the density difference in the ore affects the mill designs. Be aware that both the major vendors quote mill lengths as flange-to-flange, so you must convert your SAG mill effective grinding length to F/F by adding 2 feet to the EGL and your ball mill by adding half a foot to the effective grinding length.
2. Use the diagrams in Figure 4 & 5 and choose a point on or to the right of the diagonal line. The nominal mill diameter is given by the colour of the points, and the effective grinding length can be determined using the X-axis formula.

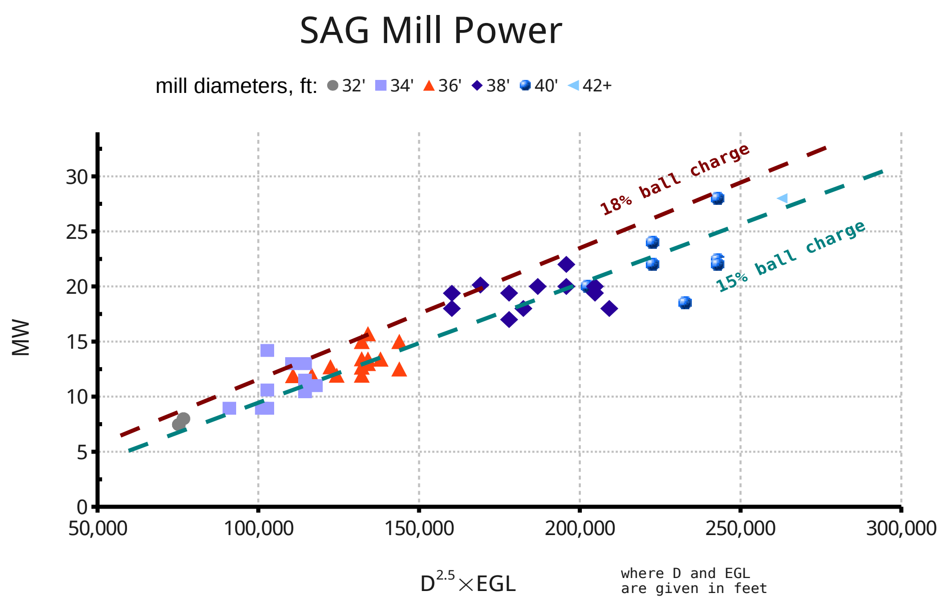

Figure 4: SAG Mill Motor Sizes Versus Function of Diameter (ft) and Effective Grinding Length (ft)[1]

Pebble Crushers

For initial design purposes, always assume a pebble crusher is required. Determine the size of the pebble crusher by choosing a crusher that can treat the volumetric flow rate equivalent to 20% of the SAG mill feed. Download vendor catalogues from their websites to see the volumetric capacities of crushers set at a 13 mm closed side setting.

If no pebble crusher is installed, Bennett *et al.*[2] suggest a SAG circuit will be 5% less efficient than a SAG circuit with a pebble crusher. Change the factor in Equation 5 from 1.10 to 1.15.

1 Doll & Barratt, *Choosing the Right Motors for your Mills,* Procemin 2010
2 Bennett, C., Dobby, G. & Kosick, G., *Benchmarking and Orebody Profiling - The Keys to Effective Production Forecasting and SAG Circuit Optimization;* SAG 2001 Conference.

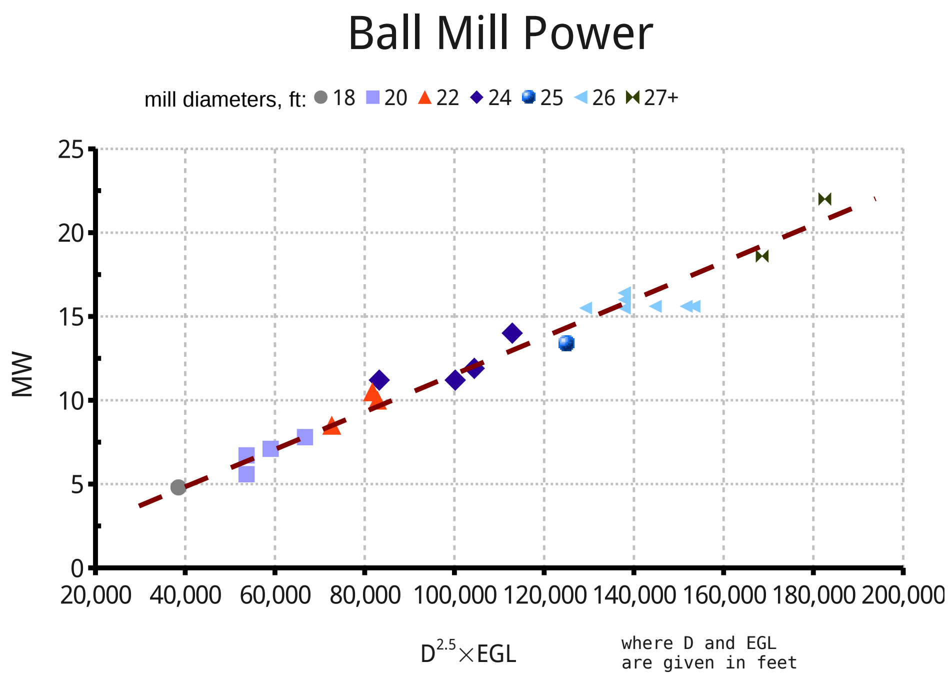

Figure 5: Ball Mill Motor Sizes Versus Function of Diameter (ft) and Effective Grinding Length (ft)

<u>Example: Tenke Fungurume</u>

Composite 1 has the following grindability results (J. Starkey et al, CMP 2007):

- Wi_{BM}=8.3 (metric)
- Wi_{RM}=12.7 (metric)
- Wi_{C}=9.0 (metric)

Estimate the specific power consumption to grind primary crusher feed of 150 mm to a flotation feed size of 150 µm. Start with the “maximum efficiency” power in Equation 4:

$$E=10\times Wi_C\left(\frac{1}{\sqrt{10000}}-\frac{1}{\sqrt{150000}}\right)+\left[10\times Wi_{RM}\left(\frac{1}{\sqrt{2100}}-\frac{1}{\sqrt{10000}}\right)+10\times Wi_{BM}\left(\frac{1}{\sqrt{150}}-\frac{1}{\sqrt{2100}}\right)\times EF5_{BM}\right]\times EF4_{BM} \qquad (11)$$

The EF4 for ball milling evaluates to 1.01, so it does apply. The EF5 for ball milling doesn't apply because the product size is coarser than 75 µm.

Equation 11 simplifies to: E = (0.67 + [1.50 + 4.97 × 1.00] × 1.01) = 7.20 kWh/tonne

Total SAG and ball mill circuit specific energy consumption: E_{total} = 7.20 × 1.10 = 7.92 kWh/tonne

Determine the SAG mill specific energy consumption using Equation 6. Recalculating the EF4 factors results in values less than 1.0 ($EF4_{RM}$=0.61, $EF4_{BM}$=0.99), use use 1.0 for both. EF5 does not apply because the product size is greater than 75 µm.

$$E_{SAG}=\left[10\times Wi_C\left(\frac{1}{\sqrt{10000}}-\frac{1}{\sqrt{F_{80}}}\right)+10\times Wi_{RM}\left(\frac{1}{\sqrt{2100}}-\frac{1}{\sqrt{10000}}\right)\times 1+10\times Wi_{BM}\left(\frac{1}{\sqrt{110}}-\frac{1}{\sqrt{2100}}\right)\times 1\times 1\right]\times 1.25$$
$$-10\times Wi_{BM}\left(\frac{1}{\sqrt{110}}-\frac{1}{\sqrt{2000}}\right)\times 1\times 1 \qquad (12)$$

Equation 12 simplifies to: E_{SAG} = (0.67 + 1.50 + 6.10) × 1.25 – 6.06 = 4.28 kWh/tonne

Determine the ball mill specific energy consumption using Equation 8:

$$E_{bm} = E_{total} - E_{SAG} = 7.92 - 4.28 = 3.64 \text{ kWh/tonne} \quad (13)$$

Determine the SAG and ball mill motor sizes required for 2000 tonnes/hour. Using Equations 9 and 10:

$$\text{Total SAG Motor Power} = (4.28 \times 2000) \div 0.985 \div 0.90 = 9\,660 \text{ kW} \quad (14)$$

$$\text{Total Ball Mill Motor Power} = (3.64 \times 2000) \div 0.985 \div 0.94 = 7\,869 \text{ kW} \quad (15)$$

Rounding off, set the SAG mill motor to 10 MW. On Figure 4, a 10 MW SAG mill motor equates to about 100,000 on the X-axis. This size motor is also in the regime of 34 foot diameter SAG mills. Use this value and the X-axis formula to determine the mill length:

$$100\,000 = 34^{2.5} \times EGL \quad (16)$$

This simplifies to $EGL = 100\ 000 \div 6741 = 14.8$

Round this off to a SAG mill effective grinding length of 15 feet. To compare to a vendor table, add 2 feet to convert the EGL to a flange-to-flange length: 17 feet.

Set the ball mill motor to 8.0 MW, and look for a motor of a similar size in the vendor literature. The vendors typically quote motor sizes in US horsepower, so look for a motor of around 10 700 hp. FLSmidth installed a 10 500 hp ball mill at Minera Los Pelambres – that is close enough. The Pelambres mill was 21 feet diameter and 33.5 feet flange-to-flange length. Convert this length to EGL by deducting half a foot, and you have a 21 feet diameter by 33 feet EGL ball mill.

End result:

- SAG mill: 34 ft diameter by 15 ft EGL with 10 MW of motor output power
- Ball mill: 21 ft diameter by 33 ft EGL with 8 MW of motor output power
- Add a pebble crusher. Metso MP800 is suitable for this duty.

Exercise #1

Tenke Fungurume Composite 2 has the following grindability results (J. Starkey et al, CMP 2007):

- WiBM=10.4 (metric)
- WiRM=13.5 (metric)
- WiC=11.10 (metric)

Determine a SAG + ball mill circuit to grind primary crusher product of 150 mm to a flotation feed size of 200 µm at 2500 tonnes/hour.

Exercise #2

Using the milling circuit you determined for Exercise #1, what would be the throughput of Composite #1 if you feed 150 mm F_{80} rock to the SAG mill and make a ball mill product P_{80} of 200 µm? Assume the SAG mill draws 90% of available power and the ball mill 94% of available power.