BOND'S WORK INDEX: WHAT IT IS AND WHAT IT ISN'T

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Abstract

Bond's work index is one of the mostly widely used metrics of mineral grindability. In spite of its ubiquity, users are often not aware of the nuances of the work index family: what are its strengths and weaknesses. Moreover, the operating work index frequently gets confused for a specific energy (SEC) consumption in documents such as National Instrument (NI) 43-101 reports. Understanding where the work index fits into the family of power-based grinding metrics will help operators correctly apply the work index and avoid making mistakes.

Understanding your ore's variation of work index by size is particularly useful for SAG mill troubleshooting and production forecasting. Examples of the variation in work index at Centerra's Mount (Mt.) Milligan mine and the implications for mill operation and design will be discussed.

A revised definition of work index is offered that makes clear the distinction between specific energy consumption and work index.

Keywords

Bond, work index, specific energy, power model

Introduction

Bond's work index model of quantifying mineral grindability was born out of the desire for a better way of scaling up laboratory grindability test results to industrial-scale grinding mills (Lynch & Rowland, 2005). The Allis-Chalmers company (now Metso) collected a great deal of operating and laboratory data during the 1930s and 1940s with the original goal of fitting the von Rittinger model of mineral breakage that had been proposed in 1867 (Bond & Maxson, 1938; Myers, Michaelson, & Bond, 1947). This effort ultimately led not to von Rittinger's model, but a new model that now bears Bond's name. Details of the derivation are discussed in the Appendix, but the key point for this discussion is that Bond's model was empirically derived, curve fitting a typical breakage characteristic from a large and diverse data set.

Method

Specific energy consumption (SEC or just *E*) is a measure of the amount of energy consumed in a comminution process, and is the power (usually expressed as kW measured at the pinion or shell of the grinding mill) divided by the tonnage processed per unit time (in modern terms, as metric tonnes per hour, or t/h). Work index is the relation between the SEC and the amount of breakage in an ore. The most common form of this relationship is given as Equation 1, and is often referred to as "Bond's equation" or "Bond's law."

$$E = 10 \times Wi \left(\frac{1}{\sqrt{P_{80}}} - \frac{1}{\sqrt{F_{80}}} \right) \tag{1}$$

Where:

1. E is the specific energy consumption in kWh/t
2. Wi is the work index (see discussion below for the units of this quantity)
3. P_{80} is the 80% passing size of the product of the comminution process in µm
4. F_{80} is the 80% passing size of the feed to the comminution process in µm

Bond's equation can be used in a number of useful ways:

1. Process and equipment design can use the equation by inputting the desired feed (F_{80}) and product sizes (P_{80}), measuring a work index value (Wi) in the laboratory and then calculating the specific energy consumption required of the process equipment (E).

   1. Different process conditions or circuits can be compared, even in the case of varying product sizes, by calculating an operating work index where the SEC (E), feed size (F_{80}), and product size (P_{80}) are measured, and the work index (*Wi*) of the operating industrial circuit is then calculated. The operating work index is useful for performing comparisons of energy efficiency that are corrected for different amounts of size reduction.

   2. Operating mines can benchmark their grinding circuit performance against laboratory measurements of varying ore grindability. Energy efficiency can be a simple ratio of the specific energy in the plant to the specific energy predicted by the work index. Different versions of the prediction exist; one example is the Bond efficiency of industrial grinding circuits (Burke, J. M. [Ed.], 2015).

TYPES OF WORK INDEX

For the purposes of this paper, it is useful to classify three types of work indices:

1. A grindability measurement made in a laboratory work index apparatus, such as a ball mill work index (Wi_{BM}) or a rod mill work index (Wi_{RM})
3. An operating work index (Wi_O): A measurement of a specific energy consumption and size reduction in an operating plant.
4. A generic work index that represents an intrinsic ore breakage characteristic that is independent of the apparatus used to determine it (Wi). This, more nebulous work index, is mostly of interest to academics and consultants who explore the details of mineral breakage.

Plant operators are mostly interested in the first two indices, as they can tell you how your mill is behaving compared to what standardized laboratory equipment would predict.

Discussion

Bond's equation is one of a much larger family of equations that includes the classical grindability models of P.R. von Rittinger and E. Kick, according to Hukki (1962). Hukki's conjecture is that the specific energy consumption of an ore changes as the particle size diminishes according to the generalized model in Figure 1. Hukki argued that the larger family of grindability models is given by a form of Charles' Equation that can be represented as a simple differential equation given as:

$$\frac{dE}{dx}=k\,x^{\alpha} \qquad (1)$$

Where:

1. *E* is the specific energy consumption in kWh/t
5. *x* is the particle size in µm
6. *K* and α are empirical constants (or functions of size, which we'll ignore for now).

One useful solution to Charles' equation is given as Equation 1:

$$E=C\left(P_{80}^{\beta}-F_{80}^{\beta}\right) \qquad (1)$$

Where:

1. *E* is the specific energy consumption in kWh/t
7. F_{80} and P_{80} are the feed and product particle sizes, respectively, in µm
8. *C* and β are empirical constants

This form is familiar, and it is reasonably easy to see Bond's Equation (Equation 1) is the special case where $\beta = -0.5$, and C = (10×*Wi*). The generic work index definition applies here, as we are discussing inherent rock characteristics.

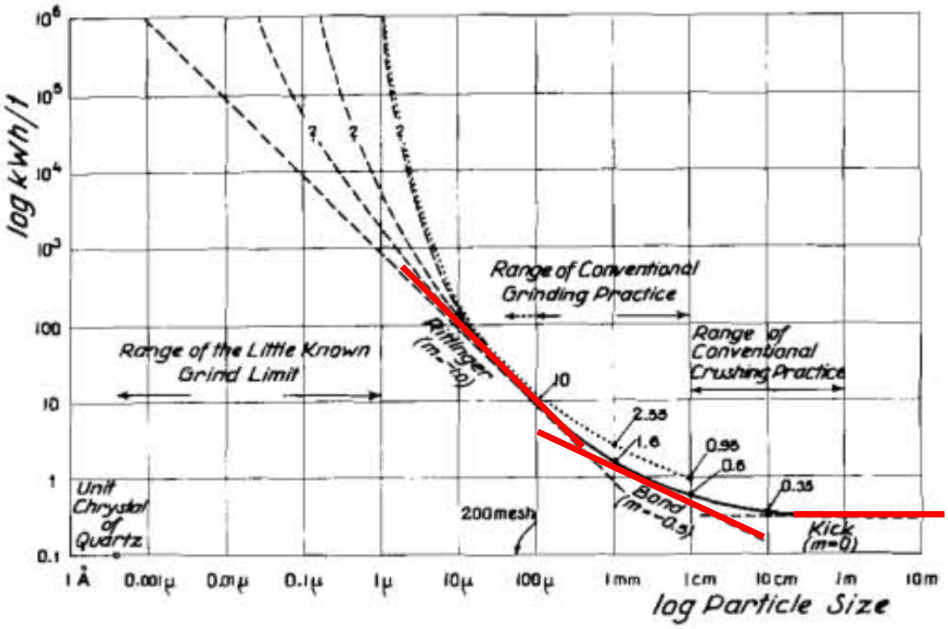

Figure 1 – Hukki's Conjecture, Cumulative Specific Energy as a Function of Particle Size

PROBLEMS WITH WORK INDEX

Work index is one of many tools used by process engineers to describe mineral comminution. Unfortunately, any tool can be used incorrectly.

There are two common problems the authors have observed in the industry. First is confusion between work index and specific energy consumption, often due to the practice of describing both work indices and specific energy in kWh/t. The second is that the inherent work index varies with particle size, and work index tests (particularly the ball mill test) can be run at the wrong closing screen size, resulting in an erroneous work index result for a particular application.

A more subtle confusion is between work index and a work index determination. The generic work index varies as a function of size, so two determinations of work index at different sizes have different results. These are not two different work indices, they are the variation in work index (singular) expressed in two different determinations (plural). To help distinguish between these cases, *work index*, without a definite or indefinite article, means the generic form; whereas, *the work index*, with a definite or indefinite article means a determination of a value for work index.

Confusion Due to kWh/t as Units of Work Index

The authors have observed two examples where authors of NI 43-101 reports (whose identity will not be disclosed, for obvious reasons) incorrectly use work index as follows:

The operating work index is calculated monthly from the kWh power consumption and actual tonnes milled.

The resulting operating work index was calculated to be in the order of 10.2 kWh/t (at pinion).

Both of these examples give numeric values that are specific energy consumption: the power draw of the mill motor divided by the throughput. Neither of these are work index values because they omitted the size reduction term of Bond's Equation. The engineers writing these reports may have been confused because the industry commonly uses units of kWh/t for both quantities, even though the two quantities refer to different things. It is incorrect to say, for example, "the ball mill work index is 13 kWh/t and the SAG mill SEC is 5 kWh/t, so the total circuit energy is 18 kWh/t." This example is a real discussion that one of the authors had with a flotation engineer who was confused because of the convention of using the same units for both quantities.

A significant goal of this paper is to avoid these mistakes in the future. To that end, the meaning and measurement units of work index will be explored with a goal of providing process engineers with the deeper understanding needed to avoid these types of mistakes. The correct units of work index are cumbersome for casual speech, so a shorthand placeholder is suggested; the actual units will be described in an upcoming section. The convention used in this paper is that only specific energy consumption has units of kWh/t, work index will instead be described using shorthand terms "metric" or "short ton basis" to make clear that it is not additive with SEC.

Work Index Changes with Particle Size

A second problem with work index is that it frequently changes as a function of size. People are occasionally erroneously taught that a homogeneous material's work index is constant. The reality is that work index is only constant for materials that have the empirical exponent of –½ when a series of work index determinations are conducted at a variety of product sizes. For example, the ball mill work index measured on a sample of ore from Mt. Milligan gave the following work index measurements (the details of the determinations are provided in the Appendix):

- Wi_{BM} = 18.3 metric units with a closing screen of 106 µm (P_{80} = 76 µm)
- Wi_{BM} = 17.5 metric units with a closing screen of 150 µm (P_{80} = 101 µm)
- Wi_{BM} = 18.2 metric units with a closing screen of 212 µm (P_{80} = 136 µm)
- Wi_{BM} = 19.4 metric units with a closing screen of 300 µm (P_{80} = 183 µm)

The reason for this variation in ball mill work index is because the ore at Mt. Milligan has an intrinsic exponent of –0.45 in the 100 µm to 200 µm size range, not the –0.50 exponent empirically measured by F. Bond. The work index increase between 100 µm and 76 µm is due to a porphyritic grain size that causes a spike in energy consumption in this size range, so the exponent changes discontinuously below 100 µm. These variations, as a function of particle size, have been long known and are the reason why the tumbling mill work index tests should be run with a closing mesh that provides the P_{80} in the laboratory that is close to the P_{80} expected in the industrial plant.

Sometimes people erroneously claim that the "10" in the work index equation, the square-root of 100 µm, means that all ball mill work index tests should be run to achieve a P_{80} of 100 µm (a closing screen of 150 µm, for example). The target work grind size at Mt. Milligan is slightly coarser than 200 µm, meaning that the work index test should be run with the 300 µm closing screen, or possibly coarser. As can be seen from the work index results above, targeting a 100 µm P_{80} will result in a highly erroneous work index determination for use calculations at the industrial plant (17.5 versus 19.4 metric units).

DEFINITION OF WORK INDEX

The framework of the Hukki Conjecture leads the authors to propose a more formal definition of (the generic) work index, albeit one that only useful for academics:

> *Work index is defined as one-tenth of the coefficient of the integrated form of the Charles Equation for the case of a fixed exponent of –0.5.*

This is not the most useful definition for plant operators, so a different definition is proposed that gives the essence of a useful definition of a work index:

> *Work index is related to the rate of change in specific energy consumption for a particular size reduction.*

In this simplified form, an operator understands that work index is not at all equal to SEC; it is more like a derivative of SEC as a function of particle size and size reduction. Ore with a larger work index will consume more specific energy for a given size reduction.

A couple of things that work index is not:

- Work index is not the SEC one observes by grinding an ore from an infinite particle size to 100 µm. This would only be true if work index was constant over the whole size range from infinity to 100 µm, and that is almost never the case for real ores.
- Work index is not a measure of the propagation of linear cracks through spherical bodies. Bond developed this notion thinking about a geometry that yields the observed exponent of -½. This was just a conjecture to explain what was observed in the Allis Chalmers data and not a definition.

THE UNITS OF WORK INDEX

Work index is commonly ascribed the units of kWh/t because of the empirical way that Bond developed the metric based on ratios of plant and laboratory data (further discussion is presented in the Appendix). In Bond's method, the 10 in the equation is the square-root of 100 µm; this was a common size that grinding circuits would grind to, and was a convenient ratio that made the square root term easy to compute.

This is an archaic definition that is solely due to Bond's empirical process and has no fundamental basis beyond Bond's ratio-based derivation. A better and more modern approach is to use work index in the context of the larger family of equations derived from Charles' Equation 1. In these terms, the 10 is vestigial and has no units; therefore, the work index has units of $\text{kW}\cdot\text{h}\cdot\text{t}^{-1}\cdot\mu\text{m}^{-0.5}$. This is an unwieldy quantity, so the authors suggest disclosure of the "ton" basis of a work index as a simple shorthand, so describe it as "metric" or "short ton basis."

Case Study, Mount Milligan

A series of work index values were determined for a sample of secondary crushed SAG mill feed at the Mount (Mt.) Milligan process plant in north-central British Columbia that was collected on December 13, 2017. A laboratory in Kamloops, BC, which has a Bond-type rod mill, provided the grindability results shown in Table 1 (see the Appendix for detailed test outputs).

Table 1 – Summary of Mt. Milligan Survey Grindability Test Results

Survey Date	Ball Milling Wi_{BM} Metric†	Rod Milling Wi_{RM} Metric	Crushing Wi_{C} Metric	Density (t/m^3)	A×b	Mia*	Mib	Ai
Dec. 13, 2017	20.2	20.4	20.1	2.8	27.3	26.9	21.4	0.06

Notes: † See discussion below
* Estimated based on JK DWT A×b

Several ball mill work index tests were conducted at different closing mesh sizes to gauge the effect of particle size versus ball mill work index (see Figure 2). The Levin "B" value (Levin, 1992) was calculated for each product P_{80} size and used in a functional performance calculation to assess the health of the ball mills (which is beyond the scope of this paper, but the observation that the Levin B value also changes with P_{80} is useful).

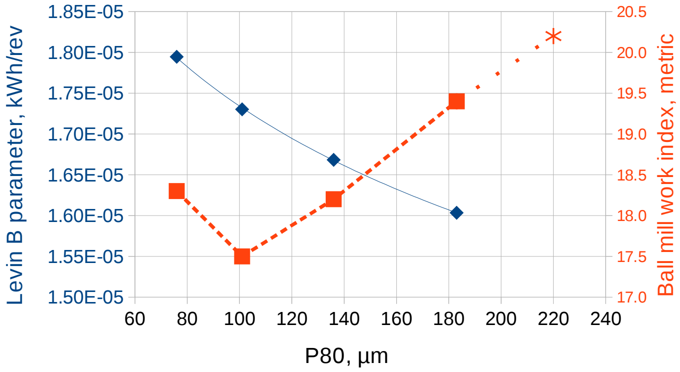

Figure 1 – Ball Mill Grindability Test Results at Different Closing Sizes

The target grind size at Mt. Milligan is 212 µm, which is considerably different to the values obtained in the ball mill grindability tests. A Josefin Equation (Josefin & Doll, 2018) was used to extrapolate the work index at the target grind size based on the ore's measured exponent of −0.45; this corrected work index was used for throughput modelling.

A different ore that has similar work index values, such as Detour Gold in Ontario (Torrealba-Vargas et al., 2015), would reasonably expect to have a similar grinding circuit configuration. An ore with dissimilar work index values, such as Antapaccay in Perú (García & Villanueva, 2013), would reasonably expect to have a different configuration. The work index values of the three deposits are given in Table 2. Mt. Milligan and Antapaccay have a coarse target P_{80}, in the range of 180 µm to 212 µm, versus a finer grind at Detour around 100 µm.

Table 1 – Grindability Test Results at Detour Gold, Mt. Milligan, and Antapaccay

	Crushing Wi	**Rod Mill Wi**	**Ball Mill Wi at P_{80}=100 µm**	**Ball Mill Wi at P_{80}=220 µm**
Detour Gold	20	17 (from A×b value)	15.5	-
Mt Milligan	20	20	17.5	20.2
Antapaccay	6	13.5	-	17.6*

Note: *P_{80} of Antapaccay samples not given; expected to be 212 µm closing screen, so P_{80} slightly less than 200 µm.

Since Detour and Mt. Milligan ores have similar work index profiles, it is reasonable to suspect they will have similar flowsheets. Sure enough, the two do have similar configurations:

- Secondary crushing of primary crushed ore; Detour uses secondary crushing on the whole SAG feed stream while Mt. Milligan can optionally secondary crush all, part, or none of the SAG feed.
- "Barely autogenous grinding" (a.k.a. BAG milling), with a high ball charge greater than 18% by volume.
- Pebble crushing with return of material to the SAG mill feed.

By contrast, Antapaccay does have the BAG mill and pebble crush return to the SAG, but does not have secondary crushing. The very low crushing work index means that secondary crushing would not contribute a lot of useful comminution energy to the ore. Secondary crushing is not useful if the coarse ore will break into medium-sized chunks the moment it is dropped into the SAG mill, which is (crudely) what the low crushing work index implies and what is observed in such plants.

BENCHMARKING PLANT PERFORMANCE TO LABORATORY RESULTS

Grindability models are calibrated empirically to match industrial plant performance to laboratory test results. The authors of grindability models carefully curate their calibration data set to exclude poorly operating industrial mills, such as those with slurry pooling or other performance problems. As such, operators can use Bond type models such as the one published by Barratt (1986) to benchmark their plant's performance against the model's calibration data set of what a healthy operating industrial plant should observe.

The work index values at Mt. Milligan, in Table 1, are reasonably constant across all the size classes (the ore's intrinsic exponent is similar to –½ across a wide size range), which mean that Bond-type models should be suitable for predicting the plant throughput. A 2017 survey measured a total grinding specific energy consumption (SAG + ball mills) of 20.8 kWh/t, and a Barratt model predicted 20.7 kWh/t; the prediction was within 0.5% of the survey. This suggests a healthy grinding circuit: the laboratory and model benchmarks are being replicated in the industrial plant.

An unhealthy circuit will consume more specific energy than what the models predict. For example, Yanacocha (Burger et al., 2011) reported "grate pegging was significantly more severe during the first survey" and "significant slurry pooling was observed in the Yanacocha SAG mill during both of the surveys." The measured specific energy consumption of the first survey was 19.8 kWh/t, 20% higher than Barratt's model prediction of 16.4 kWh/t. Given that Barratt's model is calibrated to mills operating without pegged grates and slurry pools, it is reasonable to expect that Yanacocha can achieve performance similar to Barratt's model predictions if these issues are rectified.

Conclusions

Work index is a measurement that assumes particle breakage is related to the particle size raised to a common exponent. This simplification is generally useful, but the reality is that each rock has an intrinsic exponent that must be measured. When the rock's measured exponent is different from the commonly assumed exponent, then changes in work index as a function of size will be observed, even within homogenous materials.

Work index is related to the change in specific energy consumption needed to achieve a certain size reduction. The unfortunate common practice of describing the units of work index with the same units as specific energy consumption (kWh/t) leads to confusion, and this practice should stop. Specific energy consumption should be described with units of kWh/t and work index simply as a shorthand of "metric" or "short ton basis" or use the correct units of $\mathrm{kW{\cdot}h{\cdot}t^{-1}{\cdot}\mu m^{-0.5}}$.

Operators should think of work index as the change in specific energy consumption for a particular size reduction. It is more akin to the derivative of specific energy as a function of particle size than an analog of specific energy.

Laboratory work index measurements and grindability models, when properly used, are a useful benchmark to gauge the efficiency of operating plants.

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Appendix – Details of Mt. Milligan Grindability Tests

Bond ball mill grindability test, repeated at several closing sizes

Closing Screen Aperture, P_{100}, µm	Sample Feed Size, F_{80}, µm	Sample Product Size P_{80}, µm	Grams per Mill Revolution g/rev	Bond Work Index Wi_{BM}, metric	Levin B, kWh/rev	Feed %Under-size
106	3,227	76	0.93	18.3	1.79×10^{-05}	7.80%
150	3,256	101	1.1	17.5	1.73×10^{-05}	9.00%
212	3,216	136	1.19	18.2	1.67×10^{-05}	11.60%
300	3,273	183	1.26	19.4	1.60×10^{-05}	14.20%
(300+)	*(3,256)*	*220*	*1.39*	*20.2*	*corrected to 220 µm using Josefin equation*	

Bond rod mill grindability test

Closing Screen Aperture, P_{100}, µm	Sample Feed Size, F_{80}, µm	Sample Product Size P_{80}, µm	Grams per Mill Revolution g/rev	Bond Work Index Wi_{RM}, metric
1,180	9,397	947	5.7	20.4

Bond low-energy impact crushing work index test

Quantity of Specimens	Average of Specimens, metric Wi_C	Minimum of Specimens, metric Wi_C	Maximum of Specimens, metric Wi_C	Std. Dev. of Specimens, metric Wi_C	Density, t/m³
16	20.08	4.9	42.2	10.8	2.81

JK Drop Weight Test

A	b	A×b	ta	Density, t/m³
66.5	0.41	27.27	0.21	2.80

Appendix – Non-Standard Laboratory Rod Mill Apparatus

The apparatus for determining the Bond rod mill work index is specified by the Global Mining Guidelines Group (Burke, J.M. [Ed] 2015) as follows: “The Bond rod mill is made of metal, 305 mm maximum inside diameter, with a wave-type lining. The internal mill length is 610 mm. The grinding charge consists of six 31.8 mm and two 44.5 mm diameter steel rods, all 533.4 mm in length, and weighing a total of 33,380 g. The Bond rod mill runs at 46 rpm, and has a revolution counter. In order to deal with material segregation at the ends, it is run in a level position for eight revolutions, tilted 5 degrees up for one revolution, and then tilted 5 degrees down for one revolution repeatedly during each grinding period."

There are commercial laboratories who advertise Bond rod mill work index determination using apparatus with a smooth liner, and/or where the mill is not tipped (Doll, 2016b). These non-standard machines do not return valid Bond rod mill work index values and should be avoided, except in the case where grindability models are specifically calibrated to these non-standard machines.

Doll (2016a) published a diagram, reproduced in Figure a3, that demonstrates the results of the smooth liner machines are generally higher than the equivalent results in a Bond-type apparatus, when judged against a neutral metric such as the A×b parameter from drop weight type tests.

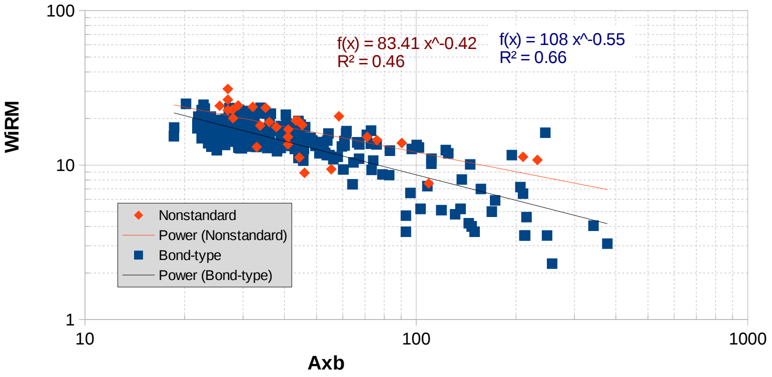

Figure a1 – Bond and Non-Standard Rod Mill Work Index vs. Drop Weight A×b

Appendix – Commentary on F. C. Bond's Work Process

Bond (1952) developed his work index based on evaluation of empirical grinding data collected by the Allis-Chalmers Company during the 1930s and 1940s. Bond plotted the cumulative specific energy consumption SEC as a function of 80% passing size, and observed that many data sets had an exponent of –½ on the size term (see Figure a4). To quote Bond (1952), "The total work useful in breakage which has been applied to a stated weight of homogeneous broken material is inversely proportional to the square root of the diameter of the product particles."

The design practice at the time of Bond's work was to scale up laboratory results using ratios of specific energy in operating plants. This is probably where $\sqrt{100}$ came from: Bond settled on a "standard" circuit where a 100 µm product size was obtained, then built up his equation from that ratio. The equation that Bond used in his 1952 Third Theory paper is given in Equation (a-1), and is not the form that is familiar to modern engineers.

$$W = Wi\left(\frac{\sqrt{F}-\sqrt{P}}{\sqrt{F}}\right)\sqrt{\frac{100}{P}} \tag{a-1}$$

Hukki (1962) observed that the "work index" is one-tenth of the coefficient of the generalized integrated form of R. J. Charles' equation (1957) where the integrated exponent is fixed at –½.

Doll, 2017, demonstrates a number of situations where the actual exponent of Charles' Equation is not -0.5, especially in the context of fine grinding. Such ores within the specified size range should be considered unsuitable for modelling by work index type models without adjustments.

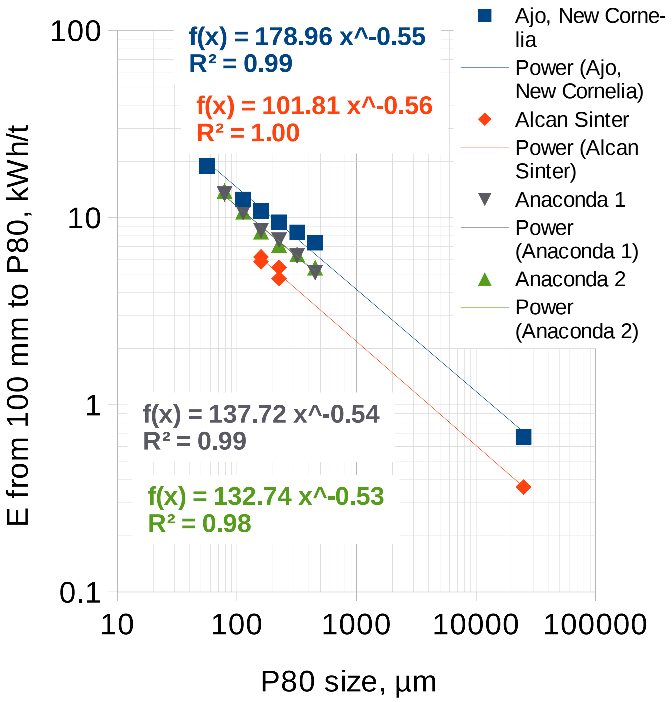

Figure a1 – Examples of Empirical Data used in F. C. Bond's Model Calibration